Improving Shell Model Calculations Using Statistical Approaches

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Outline

Neutrinoless Double-Beta Decay

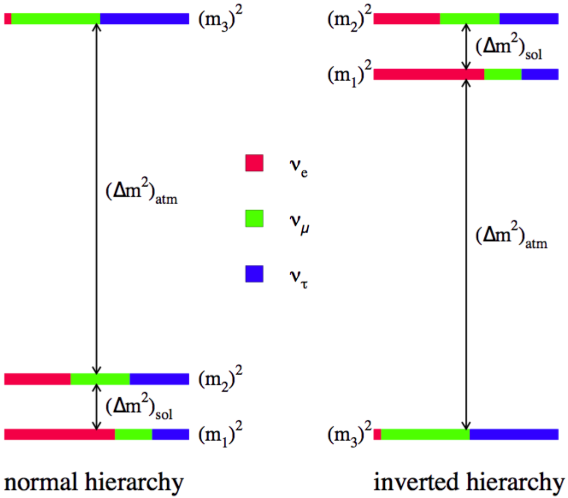
- $0\nu\beta\beta$ decay: $(A,Z) \to (A,Z+2) + 2e^-$; $\Delta L = 2$
- Forbidden in the Standard Model — requires ν to be Majorana
- If observed: unambiguous evidence of lepton number violation
- Probes: Majorana nature of neutrinos, absolute mass scale, BSM physics

Inverse half-life:

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}(Q_{\beta\beta}, Z)\, g_A^4 \left|M^{0\nu}\right|^2 \frac{\langle m_\nu \rangle^2}{m_e^2}$$

- $G^{0\nu}$: Phase-space factor (precisely known)
- $M^{0\nu}$: Nuclear matrix element (dominant uncertainty)
- $\langle m_\nu \rangle$: Effective Majorana mass (target)

Neutrino Mass Ordering and Effective Mass



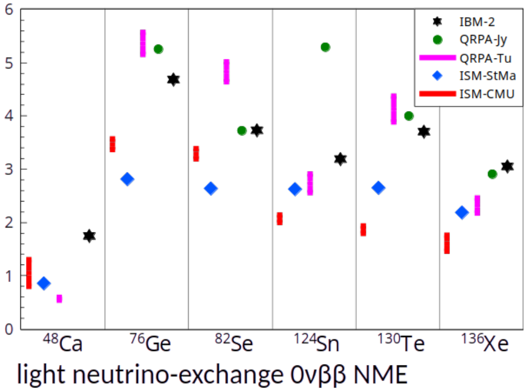
- Current best limits: $T_{1/2}^{0\nu} > 2 \times 10^{26}$ yr (^{76}Ge, ^{136}Xe)
- Next-generation "tonne-scale" experiments probe normal hierarchy
- Precise NME essential for extracting $\langle m_\nu \rangle$

The NME Problem

Many-body methods for $M^{0\nu}$:

- Interacting Shell Model (ISM)
- pn-QRPA (various versions)
- Interacting Boson Approximation (IBA-2)
- Energy Density Functionals
- Coupled Cluster
- In-Medium SRG / GCM

Factor-of-∼3 spread in NME values across methods for some isotopes



https://link.aps.org/doi/10.1103/PhysRevC.93.024308

The Shell Model Advantage

- Systematic treatment of configuration mixing
- Minimal reliance on phenomenological tuning
- Successful prediction of $2\nu\beta\beta$ half-lives before measurement
- Independent groups with different interactions produce consistent results
- Well-suited for uncertainty quantification

Goal: Quantify theoretical uncertainty of $M^{0\nu}$ within ISM using controlled statistical ensembles

Statistical Approach: Ensemble of Hamiltonians

Protocol (Horoi, Neacsu, Stoica, PRC 2022, 2023):

1. Start with 3 well-established effective Hamiltonians
2. Generate perturbed ensembles: vary two-body matrix elements (TBME) uniformly within $\pm 10\%$
3. Keep single-particle energies (SPE) fixed
4. Compute $M^{0\nu}$ and a set of low-energy observables for each sample
5. Analyze correlations and perform Bayesian Model Averaging (BMA)

Rationale:

- $\pm 10\%$ perturbation stays within empirically motivated bounds
- Prevents unphysical overfitting
- Samples the natural spread of shell-model predictions

Observables for Statistical Analysis

For each perturbed Hamiltonian, we compute:

- $M^{0\nu}$ — the target neutrinoless DBD NME
- $M^{2\nu}$ — the two-neutrino DBD NME
- $E(2_1^+), E(4_1^+), E(6_1^+)$ in parent and daughter nuclei
- $B(E2)\uparrow$ transition strengths to first 2^+ states
- Gamow-Teller strengths $B(GT)$ to first 1^+ state in intermediate nucleus
- Neutron and proton occupancies (where experimental data exist)

Benchmark: Compare ensemble distributions against experimental data to validate the approach

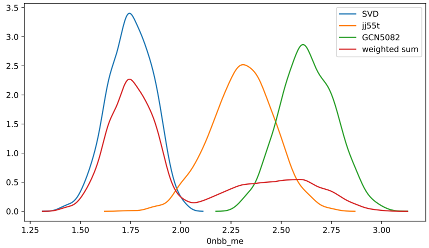
Posterior PDF:

$$P(M^{0\nu}) = \sum_k W_k \, P_k(M^{0\nu})$$

where W_k are weights from evidence integrals.

Compromise weighting:

- Prior: $W_k = 1/3$ each
- Posterior: dominated by best-performing Hamiltonian
- Final: average of prior and posterior $\rightarrow$ avoids overconfidence



^{48}Ca in the *fp* Shell

Isotope: $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ **Reference:** Horoi, Neacsu, Stoica, PRC 106, 054302 (2022)

- **Model space:** *fp* $(0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2})$
- **Starting Hamiltonians:** FPD6, KB3G, GXPF1A
- **Samples:** 20 000 perturbations per Hamiltonian
- **Key constraints:** SPE kept fixed to maintain ^{48}Ca magicity

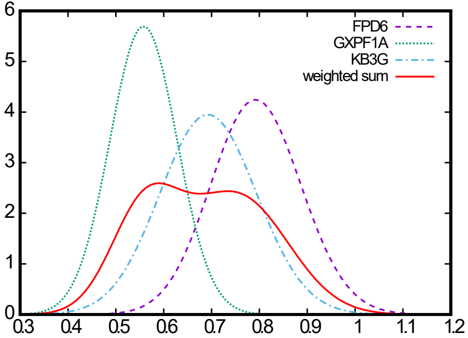
Correlation searches

Strong correlation between $M^{0\nu}$ and $M^{2\nu}$; good correlation with $E(2^+, 4^+, 6^+)$ in ^{48}Ti and neutron occupancies in ^{48}Ca.

^{48}Ca: NME Distribution

- **Mean:** $M^{0\nu} = 0.68$
- **90% C.I.:** $[0.45, 0.95]$
- Weighted sum from BMA of FPD6, KB3G, GXPF1A
- Gaussian-like distributions for all observables
- $M^{0\nu}$-$M^{2\nu}$ correlation: $\rho > 0.8$

"Reliable experimental occupation probabilities in ^{48}Ti and ^{48}Ca would further reduce uncertainties."



^{136}Xe in the *jj*55 Shell

Isotope: $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ **Reference:** Horoi, Neacsu, Stoica, PRC 107, 045501 (2023)

- **Model space:** $jj55$ $(0g_{7/2}, 1d, 2s, 0h_{11/2})$ above ^{100}Sn core
- **Starting Hamiltonians:** SVD, jj55t, GCN5082
- **Samples:** 1 000 perturbations per Hamiltonian
- **Observables:** 24 (including occupancies)
- Motivations: longest half-life limit, tonne-scale candidate

^{136}Xe: Correlations and Results

Key correlations ($|R| > 0.5$):

- $M^{0\nu}$-$M^{2\nu}$: $R \approx 0.8$ (SVD Hamiltonian)
- $M^{0\nu}$ with $E(2^+, 4^+, 6^+)$ in ^{136}Xe/^{136}Ba: $R = 0.64$–0.78
- $B(E2)$ with occupation probabilities: $|R|$ up to 0.93
- $g_{7/2}$ neutron vacancies in ^{136}Ba vs $M^{0\nu}$: $R = 0.61$

Result

$M^{0\nu} = 1.99 \pm 0.37$ **90% C.I.:** $[1.55, 2.65]$

^{82}Se in the *jj*44 Shell

Isotope: $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ **Reference:** Neacsu & Horoi, Symmetry 16, 974 (2024)

- **Model space:** *jj*44 ($0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$) above ^{56}Ni core
- **Starting Hamiltonians:** JUN45, GCN2850, JJ44b
- **Observables:** 28 (including occupancies)
- Same methodology as ^{48}Ca and ^{136}Xe
- **Result:** $0\nu\beta\beta$ NME expectation value of 3, standard deviation of 0.47, and a range of 2.55 to 3.6 at the 90% confidence level

Purpose

^{82}Se served as the validation case for applying the *jj*44 protocol to ^{76}Ge

^{76}Ge in the *jj*44 Shell

Isotope: $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$

Reference: Horoi & Neacsu (2025)

- **Model space:** $jj44$ ($f_5 p g_9$) — same as ^{82}Se
- **Starting Hamiltonians:** JUN45, GCN2850, JJ44b
- **Samples:** 200 perturbations per Hamiltonian

Why ^{76}Ge?

- Among the most promising isotopes (GERDA, LEGEND)
- Most restrictive half-life limits: $> 2 \times 10^{26}$ yr
- Next-generation tonne-scale experiments

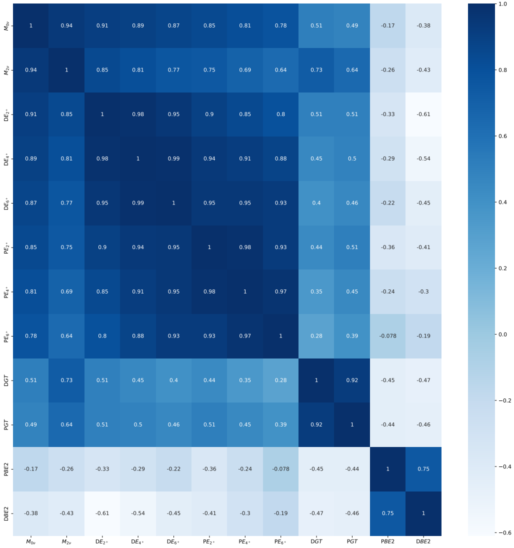
^{76}Ge: Hamiltonian Calibration

Observable	Exp. Data	GCN2850	JUN45	JJ44b
$M^{0\nu}$	—	2.389	2.701	2.534
$M^{2\nu}$	0.129(1)	0.121	0.120	0.115
$B(GT)_P$	0.120(13)	0.160	0.079	0.093
$B(E2)_P$	0.273(3)	0.242	0.250	0.280
$E(2^+)_P$ (MeV)	0.563(15)	0.718	0.745	0.718
$E(2^+)_D$ (MeV)	0.559(15)	0.511	0.574	0.669

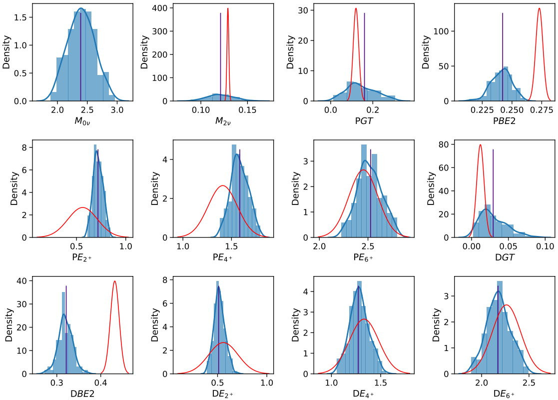
P = parent (^{76}Ge), D = daughter (^{76}Se). Energy errors: $\pm$150 keV.

^{76}Ge: Correlation Heatmap

- $M^{0\nu}$-$M^{2\nu}$ correlation: $\rho > 0.9$ (strongest link)
- Strong energy-level correlations between parent and daughter
- Anti-correlation of $B(E2)_P$ with NME and energies



^{76}Ge: Ensemble Distributions



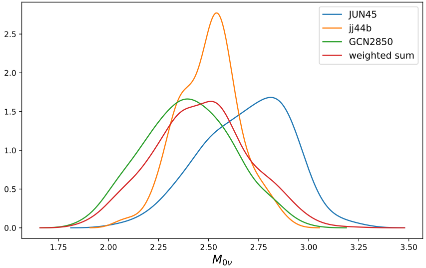
- Gaussian profiles for interesting observables — no pathological sensitivity
- Experimental benchmarks (red) generally well-reproduced
- Blue vertical bars: unperturbed Hamiltonian predictions

^{76}Ge: Bayesian Model Averaging Result

Final Result

- **Mean: $M^{0\nu} = 2.46$**
- **Std. deviation: 0.25**
- **90% C.I.: $[1.89, 3.07]$**

Weights: $W_{\text{GCN2850}} = 4/6$, $W_{\text{JUN45}} = W_{\text{JJ44b}} = 1/6$



- High consistency across all three Hamiltonians
- GCN2850 central, used as baseline

$M^{0\nu}$-$M^{2\nu}$ Correlation Across Isotopes

Isotope	Model Space	$M^{0\nu}$	$\rho(M^{0\nu}, M^{2\nu})$
^{48}Ca	*fp*	0.68 ± 0.23	> 0.8
^{136}Xe	*jj*55	1.99 ± 0.37	≈ 0.8
^{82}Se	*jj*44	3.00 ± 0.47	> 0.9
^{76}Ge	*jj*44	2.46 ± 0.25	> 0.9

Robust Feature

The strong $M^{0\nu}$-$M^{2\nu}$ correlation ($\rho > 0.8$) is universal across all studied isotopes and model spaces.

Systematic Stability Across Interactions

- *jj*44 **space (^{76}Ge, ^{82}Se):** Remarkable consistency across GCN2850, JUN45, JJ44b
- *jj*55 **space (^{136}Xe):** SVD, jj55t, GCN5082 show similar robustness
- *fp* **space (^{48}Ca):** FPD6, KB3G, GXPF1A consistent

Beyond Mass Mechanism

The half-life formula generalizes to multiple BSM mechanisms:

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 \left[|M^{0\nu}|^2 \langle m_\nu \rangle^2 + |M^{0\nu}_N|^2 \langle m_N \rangle^2 + |M^{0\nu}_\lambda|^2 \langle \eta_\lambda \rangle^2 + \cdots \right]$$

Interference effects (Ahmed, Neacsu, Horoi, PLB 769, 299, 2017):

- Mass mechanism $\leftrightarrow$ heavy neutrino mechanism
- Mass mechanism $\leftrightarrow$ η mechanism (LRSM)
- Interference can enhance or suppress decay rate
- Ratios of half-lives across isotopes can identify dominant mechanism

LRSM and Other BSM Scenarios

Left-Right Symmetric Model:

- Right-handed weak currents
- Heavy right-handed gauge bosons W_R
- New mixing parameters ζ, η, λ

Other contributions:

- Majoron emission
- SUSY R-parity violation
- Sterile neutrinos
- $0\nu\beta\beta$ as probe of GUT-scale physics

The statistical NME framework enables consistent comparison of all mechanisms.

Conclusions

1. **Robust statistical framework** for $0\nu\beta\beta$ NME uncertainty quantification in the interacting shell model
2. **Applied to four key isotopes:**
   - ^{48}Ca: $M^{0\nu} = 0.68 \pm 0.23$
   - ^{136}Xe: $M^{0\nu} = 1.99 \pm 0.37$
   - ^{82}Se: $M^{0\nu} = 3.00 \pm 0.47$
   - ^{76}Ge (new): $\mathbf{M^{0\nu} = 2.46 \pm 0.25}$
3. **Universal** $M^{0\nu}$-$M^{2\nu}$ **correlation** ($\rho > 0.8$) across all isotopes
4. **Shell model consistency** $\ll$ inter-method spread
5. **Direct application:** Theoretical neutrino mass extraction from GERDA/LEGEND limits

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Thank you!